# Functions & Calculus - Questions

\* = Question asked as part of maths quiz used at interview. Feel free to skip these ones if you

wish.

1 Functions

* 1. \* Sketch the following functions and identify their domain and range:

1. *f*(*x*) = 2
2. *f*(x) =
3. *f*(*x*) = 1/*x*
4. *f*(*x*) = loge (*x*)
5. *f*(*x*) = exp(*x*)
6. *f*(*x*) = |*x*|
7. *f*(*x*) = *x*2

Note: domain = all possible values of *x*, so for (a) domain = (-∞,∞)

range = all possible values of *f*(*x*), so for (a) range = 2

* 1. Find the roots (i.e. solutions) of the following quadratic equations:

1. \* *x*2 + *x* – 6 = 0
2. -3*x*2 - 2*x* + 1 = 0
3. *x*2 – x + 1 = 0
   1. \* Multiple choice on the exponential rules:

*a*0 = 0 1 *a* None of these

32 = 6 8 9 None of these

13 = 1 3 1/3 None of these

2-3 = -6 1/8 -9 None of these

43/45 = 48 4-8 16-1 None of these

(3-3)3 = 1 3-9 3-27 None of these

52/32 = (5/3)2 (5/3)-1 5-6 None of these

43 = 12 16 26 None of these

27-2/3 = 1/18 1/81 1/9 None of these

* 1. \* Multiple choice on the rules for logarithms:

log10(10*n*) = 10*n* *n* 10*n* 10/*n*

log10(104/10-3) = 107 1 10 7

log10(16) = 4 8 log10(2) log10(4)

log10[log10(10)] = 10 1 0 -1

log10(1000)/log10(100) = 3/2 1 -1 10

* 1. Using the rules for the summation and product functions, show that

log

(is just a quick way of saying: sum over all the *i* (i.e. in this case 1 to *n*).

2 Differentiation

* + 1. \* Differentiate the following functions:

(a) *f*(*x*) = *x*3

(b) *f*(*x*) = *x*-7

(c) *f*(*x*) = 

(d) *f*(*x*) =

(e) *f*(*x*) = exp(4*x*)

(f) *f*(*x*) = 

* + 1. Find the turning points of the following functions (that is the values of *x* at which *f*’(*x*) = 0). For each function, which is a minimum and which is a maximum?

(a) \* *f*(*x*) = 8*x* + (2/*x*)

(b) *f*(*x*) = -*x*3 - *x*2 + *x* + 3

2.3 If *L* = find

(a) (b)

(c) (d)

(e) (f)

Note that is the partial differentiation of the function *L* with respect to s. This means that we should differentiate *L* with respect to s and consider all the other variables (ie *x* and μ) in the function *L* as constant. is the second partial differentiation of *L* with respect to s. = , which means that we want to partially differentiate the function () with respect to *s*. Similarly = , which means that we want to partially differentiate the function () with respect to *s*.

3 Integration

* 1. \* Integrate the following functions:

(a) 

(b)

(c) 

3.2 Find the area under the curve for the following functions:

(a) \* y = *x*2

(i) between *x* = 0 and *x* = 3

(ii) between *x* = -3 and *x* = -0

(b) y = *x*3 – 3*x*

(i) between *x* = 0 and *x* = 3

(ii) between *x* = -2 and *x* = 2

4 Further differentiation

4.1 Differentiate the following functions:

(a)

(b)

(c)

(d)

(e)

(f)

5 Finding a maximum likelihood estimate

*X*1,...,*Xn*i.i.d. ~ N(*μ*,*σ*2), but with ***μ* known** (this means we can treat it as a constant, rather than an unknown parameter to be estimated).

Find the maximum likelihood estimate for **σ2**.

Hint: You’ll need to go through the following stages; try to keep in terms of *σ*2, not *σ*; if you have problems doing this, you can always substitute *u*=*σ*2, and substitute back at the end:

i) *L*(*σ*2|*x*) = ...

ii) *l*(*σ*2) = log(L) = ...

iii) *l’*(*σ*2) = ... [remember to differentiate with respect to *σ*2.]

iv) solve *l’*(*σ*2) = 0

Note: Normal density